

Algorithmic Trading: Order-Flow

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many thanks to

Álvaro Cartea, Oxford

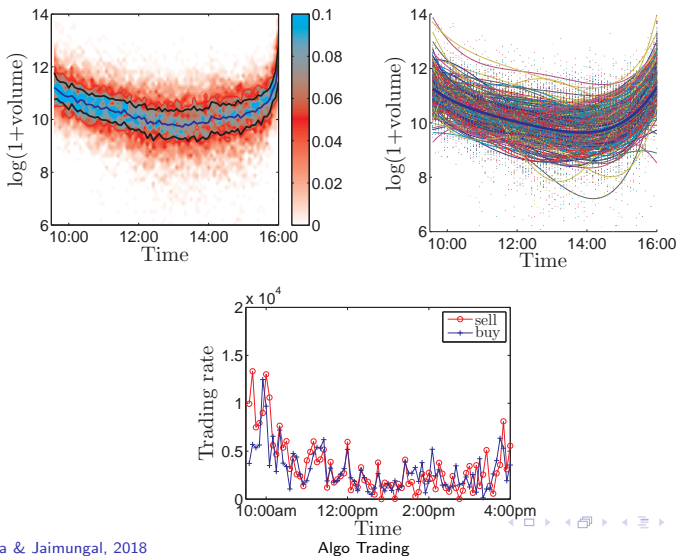
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Order-Flow

- ▶ **Trade volume** and intensity **is stochastic** throughout the day
- ▶ **All order-flow affects prices...** not just “me”
- ▶ How can we **use order-flow information** and prediction **to improve execution performance?**

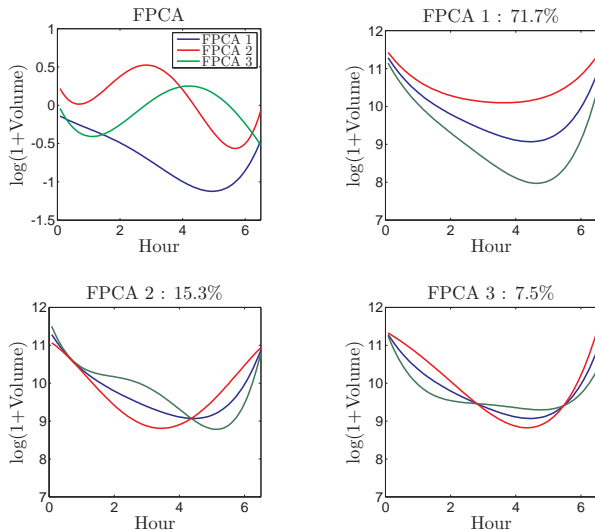
Order-Flow

Figure: ORCL (2013) traded volume using 5 minute buckets.



Order-Flow

Figure: INTL (2014) traded volume using 5 minute buckets.



Order-Flow

We assume a **linear relationship** between **net order-flow** and **midprice changes**, thus for every trading day we perform the regression

$$\Delta S_n = b \mu_n + \varepsilon_n,$$

where

- ▶ $\Delta S_n = S_{n\tau} - S_{(n-1)\tau}$ is the **change in midprice**
- ▶ μ_n is **net order-flow** defined as the difference between the volume of buy and sell MOs during the time interval $[(n-1)\tau, n\tau]$, and
- ▶ ε_n is the **error term** (assumed normal)
- ▶ In the empirical analysis we choose $\tau = 5$ min intervals.

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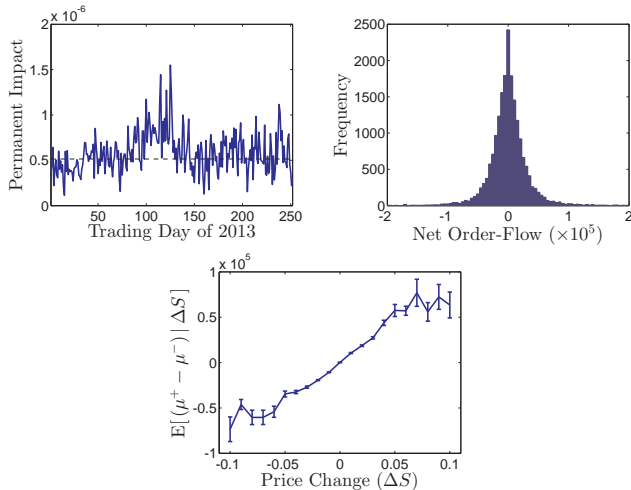


Figure: Order-Flow and effect on the drift of midprice of INTC.

Order-Flow

- ▶ Assume that **temporary price impact** is **linear** in the **trading rate** so that the **execution price** traders receive is

$$\hat{S}_t = S_t - \frac{1}{2}\Delta - k\nu,$$

- ▶ To estimate k we
 - ▶ Take snapshots of the LOB every second,
 - ▶ Determine the price per share for various volumes (by walking through the LOB),
 - ▶ Compute the difference between the price per share and the best quote at that time,
 - ▶ Perform a linear regression.

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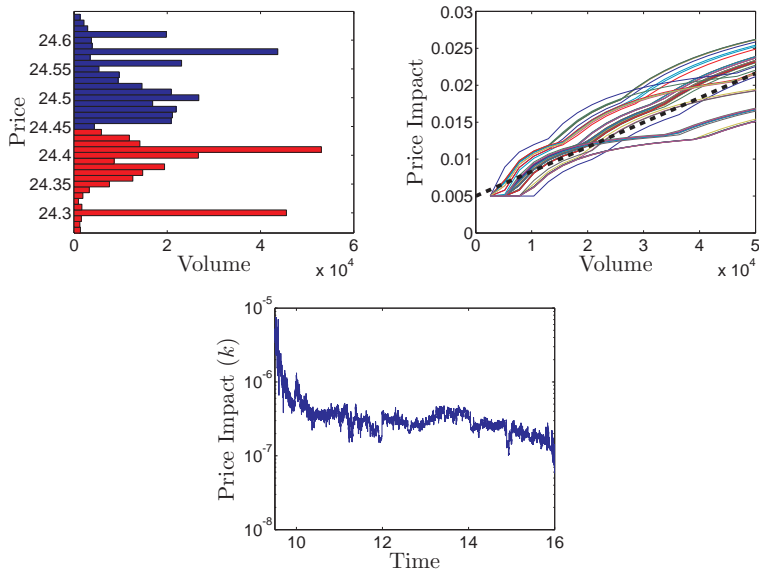


Figure: Temporary price impact: INTG on Nov 1, 2013. (a) at 11:00am, (c) Figure: Jain et al., 2008

Order-Flow

- **Temporary price impact**

$$\hat{S}_t^\nu = S_t^\nu - \frac{1}{2}\Delta - a \nu_t$$

- **Price process** is affected by order-flow from all agents, i.e., **permanent price impact**

$$dS_t^\nu = b \underbrace{(\mu_t^+ - (\nu_t + \mu_t^-))}_{\text{net order-flow}} dt + dM_t$$

- **Cash process** is

$$X_t^\nu = \int_0^t (S_u^\nu - \frac{1}{2}\Delta - a \nu_u) \nu_u du$$

Order-Flow

- ▶ Agent aims to solve for

$$H(t, x, q, S, \mu) = \sup_{\nu \in \mathcal{A}} \mathbb{E}_{t, x, q, S, \mu} \left[\underbrace{X_T^\nu + (S_T^\nu - \frac{1}{2} \Delta) q_T^\nu}_{\text{terminal book-value}} - \underbrace{\alpha (q_T^\nu)^2}_{\text{terminal inventory penalty}} \right. \\ \left. - \underbrace{\phi \sigma^2 \int_t^T (q_u^\nu)^2 du}_{\text{running inventory penalty}} \right]$$

- ▶ The DPP suggest that the value function should satisfy

$$0 = \partial_t H + \frac{1}{2} \sigma^2 \partial_{SS} H + \mathcal{L}^\mu H - \phi q^2 \\ + \sup_{\nu} \left\{ (S - \frac{1}{2} \Delta - a \nu) \nu \partial_x H \right. \\ \left. + (S + b(\mu^+ - \mu^- - \nu)) \partial_S H - \nu \partial_q H \right\}$$

subject to the terminal condition

$$H(T, x, q, S, \mu) = x + q(S - \frac{1}{2} \Delta) - \alpha q^2$$

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Proposition

Solving the DPE. *The DPE admits the solution*

$$H(t, x, S, \mu, q) = x + \textcolor{red}{q} \left(S - \frac{1}{2} \Delta \right) + \textcolor{blue}{h}_0(t, \mu) + \textcolor{red}{q} \textcolor{blue}{h}_1(t, \mu) + \textcolor{red}{q}^2 \textcolor{blue}{h}_2(t), \quad (1)$$

where

$$h_2(t) = -\sqrt{k\phi} \frac{\zeta e^{\gamma(T-t)} - e^{-\gamma(T-t)}}{\zeta e^{\gamma(T-t)} - e^{-\gamma(T-t)}} - \frac{1}{2} b, \quad (2a)$$

$$h_1(t, \mu) = b \int_t^T \left(\frac{\zeta e^{\gamma(T-u)} - e^{-\gamma(T-u)}}{\zeta e^{\gamma(T-t)} - e^{-\gamma(T-t)}} \right) \mathbb{E}_{t, \mu} \left[\textcolor{red}{\mu}_u^+ - \textcolor{red}{\mu}_u^- \right] du, \quad (2b)$$

$$h_0(t, \mu) = \frac{1}{4k} \int_t^T \mathbb{E}_{t, \mu} \left[\textcolor{red}{h}_1^2(t, \mu_u) \right] du, \quad (2c)$$

with the constants

$$\gamma = \sqrt{\frac{\phi}{k}}, \quad \text{and} \quad \zeta = \frac{\alpha - \frac{1}{2}b + \sqrt{k\phi}}{\alpha - \frac{1}{2}b - \sqrt{k\phi}}. \quad (2d)$$

Theorem

Verification. *The candidate value function is indeed the solution to the optimal control problem. Moreover, the optimal trading speed is given by*

$$\begin{aligned} \nu_t^* = & \gamma \frac{\zeta e^{\gamma(T-t)} + e^{-\gamma(T-t)}}{\zeta e^{\gamma(T-t)} - e^{-\gamma(T-t)}} Q_t^{\nu^*} \\ & - \frac{b}{2k} \int_t^T \left(\frac{\zeta e^{\gamma(T-u)} - e^{-\gamma(T-u)}}{\zeta e^{\gamma(T-t)} - e^{-\gamma(T-t)}} \right) \mathbb{E} \left[\mu_u^+ - \mu_u^- \mid \mathcal{F}_t^\mu \right] du, \end{aligned} \quad (3)$$

is the admissible optimal control we seek, and \mathcal{F}_t^μ denotes the natural filtration generated by μ .

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- ▶ If the agent requires full liquidation

$$\lim_{\alpha \rightarrow \infty} \nu_t^* = \gamma \frac{\cosh(\gamma(T-t))}{\sinh(\gamma(T-t))} Q_t^{\nu^*} - \frac{b}{2k} \int_t^T \frac{\sinh(\gamma(T-u))}{\sinh(\gamma(T-t))} \mathbb{E} \left[\mu_u^+ - \mu_u^- \mid \mathcal{F}_t^\mu \right] du.$$

- ▶ And if running penalty is absent...

$$\lim_{\phi \rightarrow 0} \lim_{\alpha \rightarrow \infty} \nu_t^* = \frac{1}{(T-t)} Q_t^{\nu^*} - \frac{b}{2k} \int_t^T \frac{(T-u)}{(T-t)} \mathbb{E} \left[\mu_u^+ - \mu_u^- \mid \mathcal{F}_t^\mu \right] du.$$

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Simulation Study

- ▶ Assume that other market participants trades follow

$$d\mu_t^+ = -\kappa^+ \mu_t^+ dt + \eta_{1+N_t^+}^+ dN_t^+,$$

$$d\mu_t^- = -\kappa^- \mu_t^- dt + \eta_{1+N_t^-}^- dN_t^-,$$

where

- ▶ $\kappa^\pm \geq 0$ are the mean-reversion rates,
- ▶ N_t^+ and N_t^- are independent homogeneous Poisson processes with intensities λ^+ and λ^- , respectively,
- ▶ $\{\eta_1^\pm, \eta_2^\pm, \dots\}$ are non-negative i.i.d. random variables with distribution function F , with finite first moment, independent from all processes.
- ▶ In addition, we require $\kappa^\pm > \lambda^\pm \mathbb{E}[\eta_1^\pm]$ is ergodic

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- ▶ The solution to order-flow is

$$\mu_s^\pm = e^{-\kappa^\pm(s-t)} \mu_t^\pm + \int_t^s e^{-\kappa^\pm(s-u)} \eta_{1+N_{u-}^\pm}^\pm dN_u^\pm,$$

so that

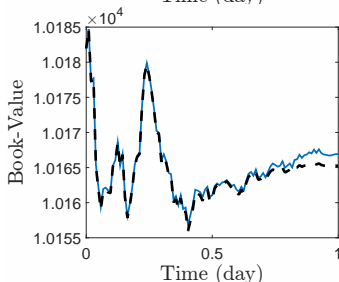
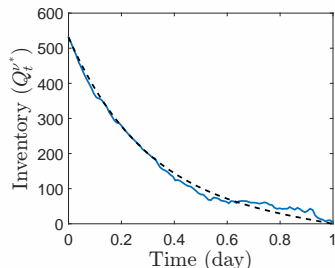
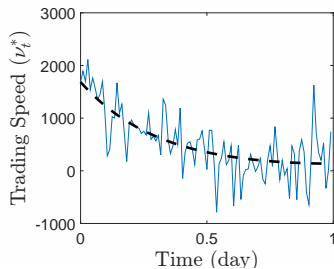
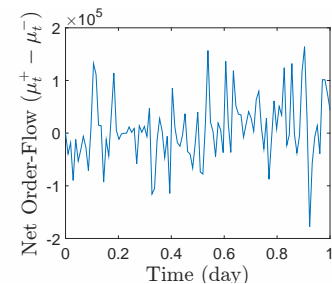
$$\mathbb{E}[\mu_s^\pm | \mathcal{F}_t^\mu] = e^{-\kappa^\pm(s-t)} (\mu_t^\pm - \psi^\pm) + \psi^\pm,$$

where $\psi^\pm = \frac{1}{\kappa^\pm} \lambda^\pm \mathbb{E}[\eta^\pm]$.

- ▶ ψ^\pm act as the expected long-run activity of buy and sell orders
- ▶ As a consequence, the optimal trading strategy is given by

$$\lim_{\alpha \rightarrow \infty} \nu_t^* = \gamma \frac{\cosh(\gamma(T-t))}{\sinh(\gamma(T-t))} Q_t^{\nu^*} - \frac{b}{2k} \left(\int_t^T \frac{\sinh(\gamma(T-u))}{\sinh(\gamma(T-t))} e^{-\kappa(u-t)} du \right) (\mu_u^+ - \mu_u^-).$$

Order-Flow



Order-flow

